A Web Application for Solving Multi-objective Integer Programming Problems

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The Research Project
The research project

- **Title:** Nondominated Points of Multi-objective Integer Programming Problems: Analysis, Approaches and Applications
- **Funding:** Scientific and Technological Research Council of Turkey (TUBITAK)
- **Members:** Murat Koksalan, Banu Lokman, Gokhan Ceyhan, Sami Ozarik, Ilgin Dogan
- **Goals:**
  - Develop *generic* algorithms to find all or representative nondominated points of *any* MOIP with *any* number of objective function
  - Provide *cloud services* to research communities by embedding these algorithms in a web application
  - Provide tools for visualization of nondominated points
- **The project is on-going.**
In development phase

Applications

- nMOCO-S: Finds all nondominated points
- rMOCO-S: Finds a representative set of nondominated points
- iMOCO-S: Integrates user preferences
- libMOCO-S: A library for MOIP instances

Technology

- Client side technologies: html5, javascript, jQuery
- Server side technologies: javaservlet
- Web server: Apache Tomcat
- Database: Apache Derby and JDBC
- Solvers: coded in C++, use IBM ILOG CPLEX API
Use case

How a user interacts with the system:
Generating all nondominated points
nMOCO-S

MOIP:

“Max” \( z(x) = \{z_1(x), z_2(x), \ldots, z_m(x)\} \)

s.t. \( x \in X \)

where \( z_i(x) \) is a continuous function of \( x \), \( X \subseteq \mathbb{R}^n \) is compact and \( x \) is a vector of \textbf{integer-valued} decision variables. We denote the image of \( X \) in the objective space as \( Z \).

- Generates all nondominated points for MOIPs
- Can also be applied as an approximate method for MOMIPs
- Based on Lokman and Koksalan (2013)
- \textbf{The idea:} The search region is partitioned and reduced progressively by removing the regions that are dominated by previously found nondominated points.
Enhancement:

- An effective data structure to handle any number of objective functions
- A smarter way of detecting the status of a search region

- N-ary tree
- **Tree height** = number of objectives ($q$) - 2
- **Number of child nodes** $\leq$ number of generated nondominated points ($N$)
Tree elements

- **Node**: defines a search space
- **Branching point**: a nondominated point already generated
- **Branching criterion**: a criterion index of the branching point
- **Bound vector**: the branching criterion values of branching points of the nodes on the path from the root node to the current node

So, at node $k$:

- $k_i$ is the branching point index at level $i$, $i = 1, 2, \ldots, q - 2$
- $z_{i_i}^k$ is the branching criterion value at level $i$
- $b^k = (z_{i_1}^k, z_{i_2}^k, \ldots, z_{i_i}^k)$ for $i = 1, 2, \ldots, q - 3$ and $b^k = (z_{i_1}^k, z_{i_2}^k, \ldots, z_{i_i}^k, z_{i_{i+1}}^k)$ for $i = q - 2$ where $z_{i+1}^{k+1} = \max_{j \in N} \left\{ z_{i+1}^j : z_{i+1}^j \geq b_{i_l}^k, l = 1, 2, \ldots, i \right\}$
Special structure

- \( z_q^1 \geq z_q^2 \geq \ldots, z_q^N \)
- Let \( C^k \) be the child nodes of node \( k \). Then,
  \[
  S^k = \bigcup_{c \in C^k} S^c
  \]
- Let \( L^k \) and \( R^k \) be the left and right siblings of node \( k \). For nodes except the leaf nodes:
  \[
  S^l \subseteq S^k \subseteq S^r, l \in L^k, r \in R^k
  \]
- At leaf nodes,
  \[
  b_{q-1}^l \geq b_{q-1}^k \geq b_{q-1}^r, l \in L^k, r \in R^k
  \]
What kind of problems can nMOCO-S solve?

- For MOIPs, any problem for which there are effective algorithms to solve single objective case
- We use IBM ILOG CPLEX as the single objective solver
  - MILP
  - MIQP
  - MIQCP
- For MOMIPs, our algorithm can generate a set of nondominated points which $\epsilon-$ dominates the nondominated set
Generating representative nondominated points
• **The idea:** Find a subset of nondominated points that *represents* the nondominated frontier well enough.

• Based on Ceyhan et al. (2014)

• **How to represent:** *Masin ve Bukchin (2008): Coverage gap*

\[
\alpha = \max_{z \in ND} \left\{ \min_{y \in R} \left\{ \max_{1 \leq i \leq p} z_i - y_i \right\} \right\}
\]

**Corollary**

If the coverage gap of subset \( R \) is \( \alpha^* \), then \( R \alpha^* \)—dominates all nondominated points. \( \forall z \in ND, \exists y \in R : z_i \leq y_i + \alpha^*, \forall i = 1, ..., p. \)
Approach 1:

- Find worst covered nondominated point at each iteration which gives the coverage gap of the available subset
- Stop when the desired coverage gap is achieved

The procedure:

- Define coverage gap as an additional objective function to be maximized
- Use nMOCO-S with $q + 1$ objective functions
rMOCO-S

Approach 2:

- Identify the desired coverage gap, $\Delta$
- Find a set of nondominated points such that $\alpha \leq \Delta$

The procedure:

- Utilize $\Delta$ actively throughout the algorithm
- Construct territories around the previously generated points that are inadmissible for the new point

- Tighten the bounds in the leaf nodes of nMOCO-S tree with $\Delta$
New features:

- Minimize density weighted coverage gap
- Try to achieve a coverage gap with the smallest possible set
  - Criteria for sorting the search spaces
  - Scalarization of objective functions for the search problem
Visualization of nondominated points
The needs:

- To capture the global characteristics of the nondominated frontier

- To investigate the local trade-offs

Figure 7: Three dimensional $L_p$ surfaces with different $p$ values
The challenges:

- There can be too many points to visualize
- More than three criteria
We developed *shiny* web application in *R* language

- We use a graphics library offered by *plotly*
- Source code is available in our *github* account
- Users should create an account in *shiny.io* and deploy our source code
Demo
Demo: How to solve a MOKP instance with nMOCO-S?
Summary
A need for publicly available multi-objective solvers
There are recent studies on this direction (e.g. vOpt-Solver)
We try to contribute to this need by providing our MOIP solvers on the web as a cloud-service
A collection of MOIP instances that could be used for benchmarking
A source code for tools of visualization
Questions?

http://onlinemoco.com/MOIP/
https://github.com/gokhanceyhan
https://onlinemoco.shinyapps.io/gMOCO/